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HEAT AND MASS TRANSFER IN MIXED CONVECTION STAGNATION POINT FLOW TOWARDS A STRETCHING SHEET IN THE PRESENCE OF RADIATION AND HEAT SINK WITH VARIABLE THERMAL CONDUCTIVITY

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ABSTRACT

The analysis of laminar mixed convection in boundary layers adjacent to a vertical, continuously stretching sheet in the presence of radiation and heat sink has been studied. It has been assumed that thermal conductivity is varying linearly with temperature. The stretching velocity, surface temperature and surface concentration varying linearly with the distance from stagnation point. The governing flow equations have been transformed into set of coupled ordinary differential equations using similarity transformation. These equations have been solved by using Runge-Kutta method with shooting technique. The effects of Prandtl number, Schmidt number, sheet stretching parameter, heat source or sink parameter and thermal conductivity parameter, on the boundary layer velocity, temperature and concentration profiles have been discussed in detail for both the cases of assisting and opposing flows. The results show that magnitude of temperature increases with increasing values of thermal conductivity parameter and velocity and temperature both increase with increasing values of heat sink parameter. Also magnitude of heat transfer coefficient decreases with increasing thermal conductivity and with heat sink parameter for assisting as well as opposing flow.

Key words : Stretching sheet, variable thermal conductivity, heat source, radiation, stagnation point, shooting technique.

INTRODUCTION

The flow over a stretching sheet is an important problem in engineering processes with applications in industries such as in hot rolling, wire drawing, crystal growing, continuous casting,

glass fiber production, extrusion of plastics and paper production. The study of heat and mass transfer is necessary for determining the quality of the final products of such processes.

The flow near a stagnation point has attracted many investigators during the past several years. In 1988 Karve and Jaluria presented flow and mixed convection transport from a moving plate in rolling and extrusion processes and in 1991 they done the numerical simulation of thermal transport with a sheet in material processing. Mahapatra and Gupta studied the heat transfer in the steady two-dimensional stagnation point flow of an incompressible fluid over a stretched sheet with a velocity proportional to the distance from the stagnation point, also with effect of viscous dissipation of the fluid in 2002. Mixed convection boundary layer flow in the stagnation point flow toward a stretching sheet is presented by Ishak in 2006. The boundary layer flow for an incompressible viscous fluid near a stagnation point at a heated stretching sheet placed in a porous medium, using Lie-group method is analyzed by Boutros. In 2007 El-Aziz studied the heat and mass transfer of electrically conducting fluid having temperature dependent viscosity and thermal conductivity along a stretching sheet with the effect of Ohmic heating. In 2007 Mahapatra and Dholey studied the magneto hydrodynamic fluid flow towards a stretching sheet. In 2009 Jing ZHU found the analytical solution to a stagnation point flow and heat transfer over a stretching sheet using Homotopy analysis. Recently in 2009 D. Pal studied effect of thermal radiation on heat and mass transfer stagnation point flow towards a stretching sheet.

In all the previous studies in mixed convection flow towards a stretching sheet, the effect of heat source with radiation and variable thermal conductivity is not studied. Aim of this paper is to study the effects of variable thermal conductivity and heat source on steady two dimensional mixed convection stagnation point flow towards a stretching sheet in the presence of radiation effect. It is assumed that the stretching velocity, surface temperature and surface concentration vary linearly with the distance from stagnation point. The governing fluid flow equations are transformed into non dimensional form using similarity transformation and solved by using Runge- Kutta method with shooting technique.

Mathematical Formulation of Problem :

The mixed convection, two-dimensional, laminar, boundary layer flow due to a stretching vertical heated sheet in a viscous, incompressible and electrically conducting fluid in the presence of heat source and radiation with variable thermal conductivity has been made. The stationary coordinate system has its origin located at the centre of the sheet with the x-axis extending along the sheet, while the y-axis is measured normal to the surface of the sheet and in the positive direction from the sheet to the fluid. The coordinate system has been shown in adjacent figure. The stretching velocity $u_w(x)$, surface temperature $T_w(x)$ and surface concentration $C_w(x)$ vary linearly with the distance from stagnation point O. The governing fluid flow equations for the problem under consideration are:

Equation of continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

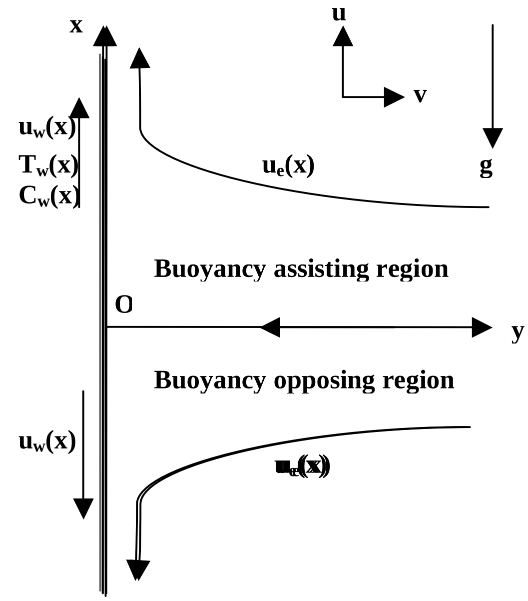

Figure (a)

Equation of momentum,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+v\frac{\partial^2 u}{\partial y^2}\pm g\beta_T(T-T_\infty)\pm g\beta_C(C-C_\infty) \qquad (2)$$

Equation of heat transfer,

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(k'\frac{\partial T}{\partial y}\right)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{Q}{\rho C_p}(T-T_\infty) \qquad (3)$$

Equation of mass transfer,

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial z^2} \qquad (4)$$

where u and v are velocity component along x and y directions. g is acceleration due to gravity, T and C are temperature and concentration of fluid, T_∞ and C_∞ are temperature and concentration at outer side of boundary layer, k' is variable thermal conductivity of fluid, Q is heat source parameter and D is mass diffusion coefficient. In equation (2) "+" sign is corresponding to assisting flow and "-"sign refers to an opposing flow. The stretching surface has temperature T_w, concentration C_w and the free stream temperature and concentration are T_∞ and C_∞ respectively, with $T_w > T_\infty$ and $C_w > C_\infty$. In this study the radiation heat flux in the x-direction has also been considered. Using Rosseland diffusion approximation the radiation heat flux is given by:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \qquad (5)$$

Keeping the temperature difference within the flow is assumed to be sufficiently small, so that T^4 may be expressed as a linear function of temperature T, i.e.

$$T^4 \cong 4T_\infty^3 T-3T_\infty^4 \qquad (6)$$

The initial and boundary conditions are:

$u=u_w(x)=U_0x$, $v=0$, $T=T_w(x)=T_\infty+bx$, $C=C_w(x)=C_\infty+dx$, at $y=0$

$u=u_e(x)=ax$, $T\to T_\infty$, $C\to C_\infty$, as $y\to\infty$ (7)

Here U_0, a, b, d are constants. u_e is velocity of the flow external to the boundary layer.
The similarity transformations used are:

$$\psi=(U_0 v)^{\frac{1}{2}}x\,f(\eta),\ \eta=\left(\frac{U_0}{v}\right)^{\frac{1}{2}}y \qquad (8)$$

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x}\ \text{ and }\ k'=k\,(1+\varepsilon\theta)$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

here ψ is stream function, η is similarity variable, ε is thermal conductivity parameter, k is thermal conductivity of fluid, θ and ϕ are dimensionless temperature and concentration. Using above transformations equations (2) - (4) are converted into following set of ordinary differential equations:

$$f'''+\left(\frac{a}{U_0}\right)^2-f'^2+ff''\pm\lambda\theta\pm\delta\phi=0 \quad (9)$$

$$(1+\varepsilon\theta)\theta''+\tfrac{4}{3}Rd\,[1+(\theta_w-1)\theta]^3\theta''+\varepsilon\theta'^2+4Rd[1+(\theta_w-1)\theta]^2\theta'^2(\theta_w-1)+Pr[f\theta'-f'\theta'+S\theta]=0 \quad (10)$$

$$Sc(f\phi'-f'\phi)+\phi''=0 \quad (11)$$

where Sc is Schmidt number, Ec is Eckert number, λ is thermal buoyancy parameter, δ is solutal buoyancy parameter.

The transformed boundary conditions are:

$$f'(0)=1,\ f(0)=0,\ \theta(0)=1 \text{ and } \phi(0)=1 \text{ at } \eta=0$$
$$f=\tfrac{a}{U_0},\quad \theta=0 \text{ and } \phi=0 \quad \text{as} \quad \eta\to\infty \quad (12)$$

Where $\alpha=\frac{k}{\rho C_p},Re_x=\frac{u_w x}{v}$ and $\theta_w=\frac{T_w}{T_\infty},Rd=\frac{4\sigma^*}{k}\frac{T_\infty^3}{k^*}$

$Gr_C=\frac{g\beta_C(C_w-C_\infty)x^3}{v^2}$ and $Gr_T=\frac{g\beta_T(T_w-T_\infty)x^3}{v^2}$ are local Grashof number and local solutal Grashof number respectively. Also $\lambda=\frac{Gr_T}{Re_x^2}$ and $\delta=\frac{Gr_C}{Re_x^2}$ are thermal and solutal buoyancy parameters. The physical quantities of practical interest are skin friction coefficient C_f, the local Nusselt number Nu_x and the local Sherwood number Sh_x. These quantities are given as

$$C_f=\frac{\tau_w}{\rho u_w^2},Nu_x=\frac{xq_w}{k'(T_w-T_\infty)},Sh_x=\frac{xq_m}{D(C_w-C_\infty)} \quad (13)$$

Here

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$q_w=-\left(\left(\frac{16\sigma^*T^3}{3k^*}+k'\right)\frac{\partial T}{\partial y}\right)_{y=0} \quad (14)$$
$$q_m=-\left(D\frac{\partial C}{\partial y}\right)_{y=0}$$

Now after using (9) and (14), we obtain

$$C_f\,Re_x^{1/2}=f''(0),\frac{Nu_x}{\sqrt{Re_x}}=-\frac{(1+\varepsilon+\frac{4}{3}Rd\theta_w^3)\theta'(0)}{(1+\varepsilon\theta)},$$
$$\frac{Sh_x}{\sqrt{Re_x}}=-\phi'(0) \quad (15)$$

RESULTS AND DISUCUSSION

The Runge-Kutta method with shooting technique has been used to solve the set of coupled ordinary differential equations $\Delta\eta=0.01$ and $\eta_\infty=8$. These results show good agreement with Pal (2008) for $a/U_0=1$, $Pr=0.72$, $\theta_w=1.1$, $\lambda=1$, $Rd=1$, $\delta=0.5$, $Sc=0.5$, $S=0$ and $\varepsilon=0$. The value of skin friction coefficient $f''(0)$ in the paper by Pal is 0.64268 for assisting flow and in the present study the value is of $f''(0)$ for same set of parameters is 0.642959.The value of dimensionless velocity $f''(\eta)$, dimensionless temperature $\theta(\eta)$ and dimensionless concen-

tration $\Phi(\eta)$ have been computed for different values of dimensionless parameters and shown from figures (1) - (10). These results are observed for both assisting as well as for opposing flow. Figure (1) presents the temperature profile with varying values of thermal conductivity parameter (ε) for assisting and opposing flow. It has been observed that magnitude of temperature increases with increasing values of thermal conductivity parameter. Figures (2) and (3) present the effect of Prandtl number on velocity and temperature of fluid. These figures show that magnitude of velocity and temperature both decreases with increasing values of Prandtl number (Pr=0.7, 5, 10). In figures (4) and (5) effect of heat source parameter on velocity and temperature have been shown. It has been seen that magnitude of velocity and temperature both increase with increasing values of heat source parameter. The effects of Schmidt number on velocity and concentration profiles have been shown in figures (6) and (7). These figures show that magnitude of velocity as well as the magnitude of concentration both decrease with increasing values of Schmidt number (Sc=0.5, 2, 5). From figures (8) - (10) effect of stretching velocity parameter (a/U_0 =0.5, 1, 1.5, 2) on velocity, temperature and concentration of fluid have been presented. In figure (8) it has been seen that magnitude of velocity increases with increasing values of stretching velocity parameter. In figures (9) and (10), it has been seen that magnitude of temperature and concentration both decrease with increasing values of stretching velocity parameter.

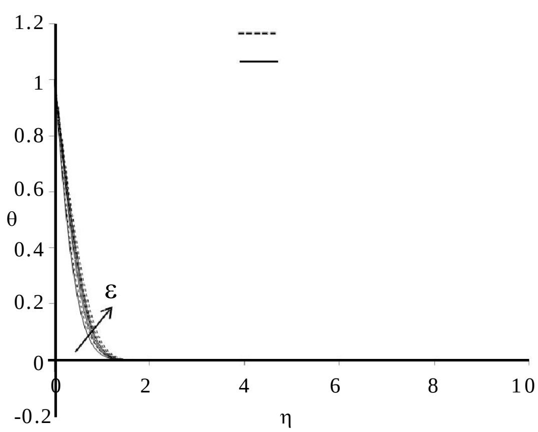

Figure 1. Temperature profile with thermal conductivity parameter
($Pr=7$, $a/U_0=1$, $S=0.1$, $Rd=0.5$, $\lambda=-1$, $\delta=0.5$, $Sc=0.5$, $\theta_w=1$)

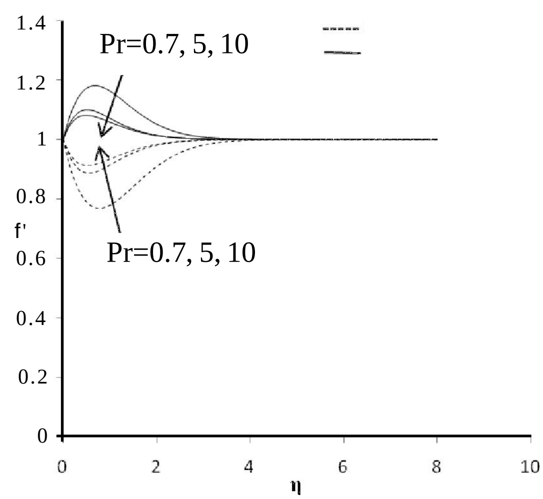

Figure 2. Velocity profile with Prandtl number
($\varepsilon=1$, $a/U_0=1$, $S=0.1$, $Rd=0.5$, $\lambda=-1$, $\delta=0.5$, $Sc=0.5$, $\theta_w=1$)

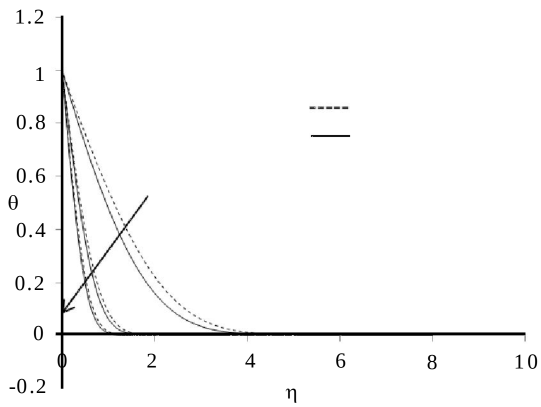

Figure 3. Temperature profile with prandtl number
(ε=1, a/U_0=1, S=0.1, Rd=0.5, λ=-1, δ=0.5, Sc=0.5, θ_w=1)

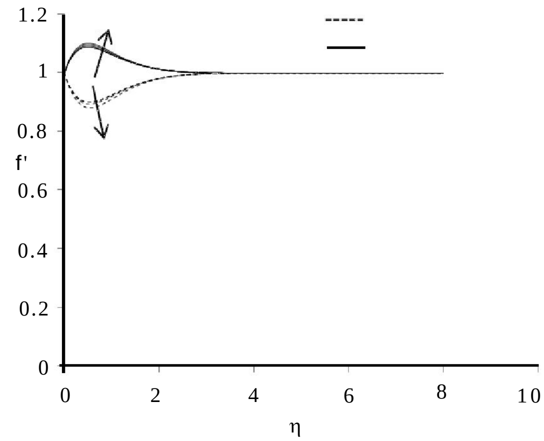

Figure 4. Velocity profile with heat source parameter
(ε=1, a/U_0=1, Pr=7, Rd=0.5, λ=-1, δ=0.5, Sc=0.5, θ_w=1)

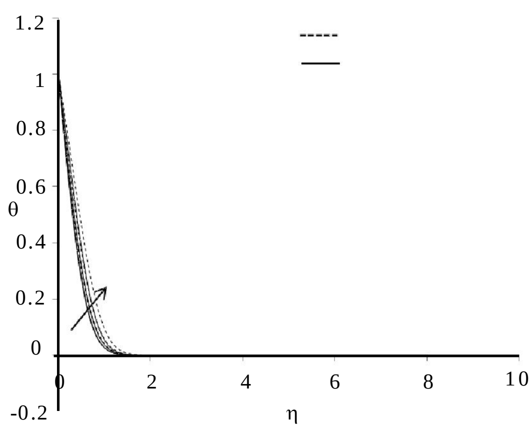

Figure 5. Temperature profile with heat source parameter
(ε=1, a/U_0=1, Pr=7, Rd=0.5, λ=-1, δ=0.5, Sc=0.5, θ_w=1)

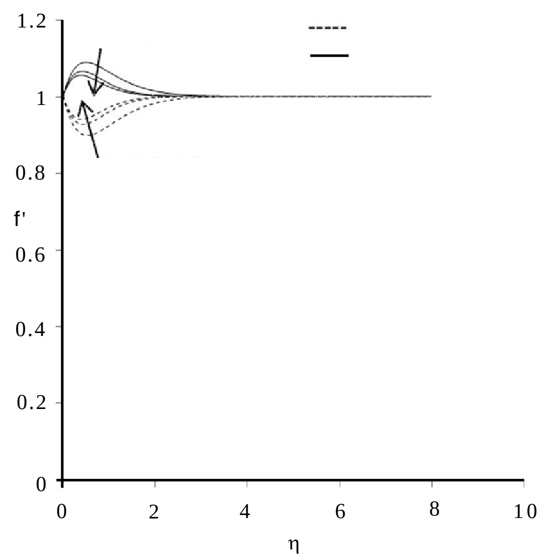

Figure 6. Velocity profile with Schmidt number
(ε=1, a/U_0=1, Pr=7, Rd=0.5, λ=-1, δ=0.5, S=0.1, θ_w=1)

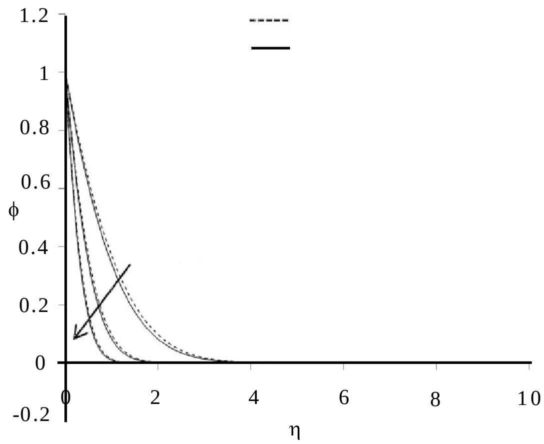

Figure 7. Concentration profile with Schmidt number
(ε=1, a/U_0=1, Pr=7, Rd=0.5, λ=-1, δ=0.5, S=0.1, θ_w=1)

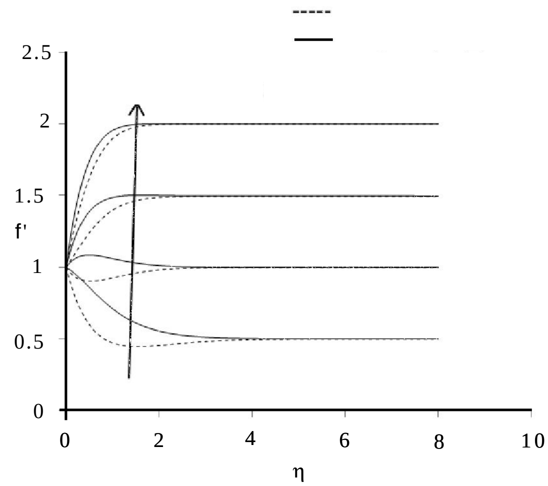

Figure 8. Velocity profile with Strerching velocity parameter
(ε=1, Sc=0.5, Pr=7, Rd=0.5, λ=-1, δ=0.5, S=0.1, θ_w=1)

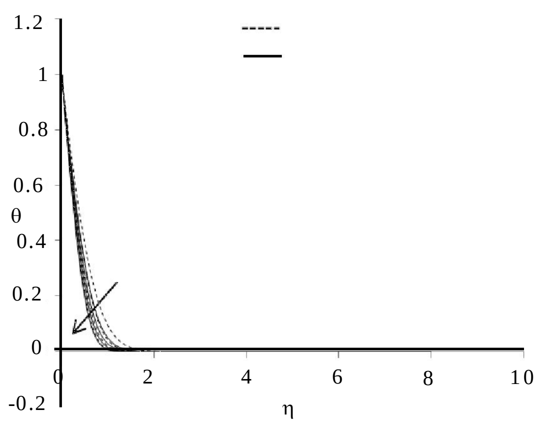

Figure 9. Temperature profile with Strerching velocity parameter
(ε=1, Sc=0.5, Pr=7, Rd=0.5, λ=-1, δ=0.5, S=0.1, θ_w=1)

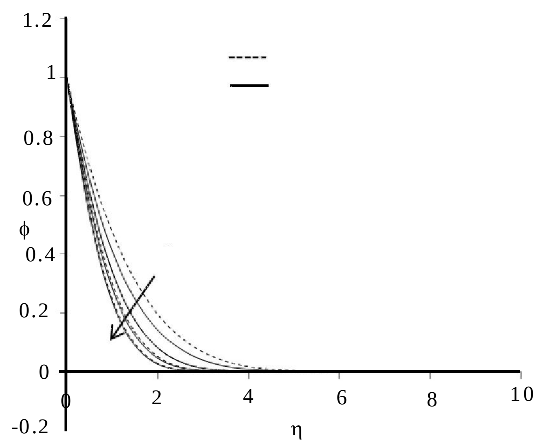

Figure 10. Concentration profile with Strerching velocity parameter
(ε=1, Sc=0.5, Pr=7, Rd=0.5, λ=-1, δ=0.5, S=0.1, θ_w=1)

The values of skin friction coefficient, heat transfer coefficient and mass transfer coefficient for assisting and opposing flow have been given in table 1 and 2, for different values of dimensionless parameters.

Table 1. Values of skin friction coefficient, heat transfer coefficient and mass transfer coefficient with different values of dimensionless parameters for assisting flow

$\lambda=1, \delta=0.5, \theta_w=1.1, Rd=0.5$							
Pr	Sc	S	a/U_0	ε	f''(0)	θ'(0)	ϕ'(0)
0.7	0.5	0.1	1	1	0.650039	-0.58543	-0.93701
7	1	0.1	1	1	0.476752	-1.51852	-0.91494
7	5	0.1	1	1	0.41772	-1.78382	-1.28459
7	0.5	0.1	1	1	0.352132	-1.77387	-2.84539
7	0.5	0.5	1	1	0.463501	-1.55798	-0.91347
7	0.5	1	1	1	0.486871	-1.22499	-0.91543
7	0.5	0.1	0.5	1	-0.161044	-1.68675	-0.81306
7	0.5	0.1	2	1	2.389276	-2.01820	-1.09576
7	0.5	0.1	1	2	0.470360	-1.49734	-0.91407

Table 2. Values of skin friction coefficient, heat transfer coefficient and mass transfer coefficient with different values of dimensionless parameters for opposing flow

$\lambda=1, \delta=0.5, \theta_w=1.1, Rd=0.5$							
Pr	Sc	S	a/U_0	ε	f''(0)	θ'(0)	ϕ'(0)
7	0.5	0.1	1	1	-0.476331	-1.68752	-0.85585
7	5.0	0.1	1	1	-0.366038	-1.70709	-2.75523
0.7	0.5	0.1	1	1	-0.73360	-0.50304	-0.81615
10	0.5	0.1	1	1	-0.443503	-2.02828	-0.85906
7	0.5	0	1	1	-0.471969	-1.74698	-0.85625
7	0.5	1.0	1	1	-0.533382	-1.02939	-0.85023
7	0.5	0.1	2	1	1.636983	-1.96508	-1.06844
7	0.5	0.1	1	0	-0.444990	-2.22646	-0.85848
7	0.5	0.1	1	2	-0.501742	-1.40201	-0.85339

In this study, it has been seen that magnitude of skin friction coefficient increases with thermal conductivity parameter, heat source parameter and also with stretching velocity parameter for both assisting and opposing flow. But magnitude of skin friction coefficient decreases with increasing values of Prandtl number and Schmidt number for both observed cases. Further it has been noticed that magnitude of heat transfer coefficient decreases with increasing thermal conductivity parameter and with heat source parameter for assisting as well as opposing flow. It has also been observed that with increasing Prandtl number, Schmidt number and with stretching velocity parameter, the magnitude of heat transfer coefficient increases. In this study magnitude of mass transfer coefficient increases with stretching velocity parameter and with Schmidt number for both observed cases and there is no significant change in heat transfer coefficient with other parameters.

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